A SPECIAL CASE OF THE STAHL CONJECTURE

J. KINCSES, G. MAKAY, M. MARÓTI, J. OSZTÉNYI AND L. ZÁDORI

Abstract. Let $G_{n,k}$ denote the Kneser graph whose vertices are the n-element subsets of a $(2n+k)$-element set and whose edges are the disjoint pairs. In this paper we prove that for any non-negative integer s there is no graph homomorphism from $G_{4,2}$ to $G_{4s+1,2s+1}$. This confirms a conjecture of Stahl in a special case.

1. Introduction

Let $G_{n,k}$ denote the Kneser graph whose vertices are the n-element subsets of the set $\{1,\dots,2n+k\}$ and whose edges are the disjoint pairs. We are interested in the question that for which values of the parameters n, k, n' and k' there exists a graph homomorphism from $G_{n,k}$ to $G_{n',k'}$. We write $G_{n,k} \to G_{n',k'}$ to denote that there exists a graph homomorphism from $G_{n,k}$ to $G_{n',k'}$.

It should be obvious that if there exist homomorphisms from $G_{n,k}$ to $G_{n',k'}$ and $G_{n'',k''}$, then there is one from $G_{n,k}$ to $G_{n'+n'',k'+k''}$. This property is called additivity of homomorphisms between Kneser graphs. The existence of homomorphisms between certain Kneser graphs is well known, see [5].

Lemma 1. *We have that* $G_{n,k} \to G_{ns,ks}$, $G_{n,k} \to G_{n-1,k}$ *and* $G_{n,k} \to G_{n,k+1}$. *Hence, if* $\frac{k'}{k} \geq \lceil \frac{n'}{n} \rceil$, *then* $G_{n,k} \to G_{n',k'}$.

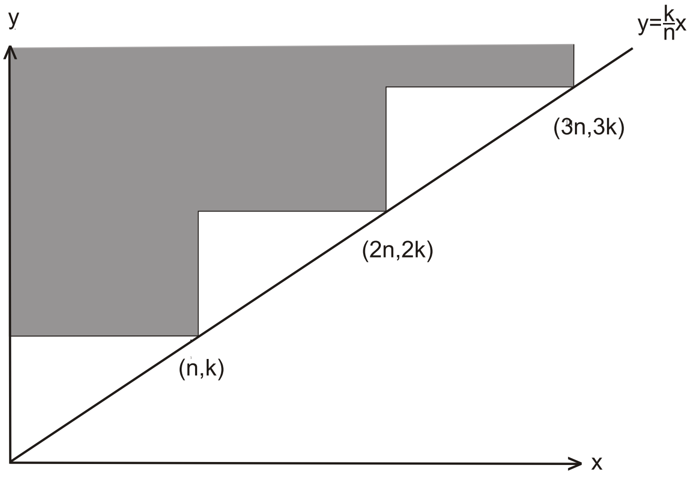

Figure 1

The meaning of the lemma is that for any grid point (n',k') in the shaded area of Figure 1 there is a homomorphism from $G_{n,k}$ to $G_{n',k'}$.

The authors' research was partially supported by the OTKA grants K77409, K83219.

The following conjecture was formulated by Stahl in [5] in a slightly different form.

Conjecture 2. *If* $\frac{k'}{k} < \lceil \frac{n'}{n} \rceil$ *then* $G_{n,k} \not\to G_{n',k'}$.

The conjecture tells us that for any pair (n', k') in the light area of Figure 1, $G_{n,k} \not\to G_{n',k'}$. In view of Lemma 1 to settle the conjecture it suffices to prove that $G_{n,k} \not\to G_{ns+1,ks+k-1}$ for all s. In fact, it suffices to prove this for arbitrarily large values of s. Indeed, for any $s' < s$, $G_{n,k} \to G_{ns'+1,ks'+k-1}$ would imply $G_{n,k} \to G_{ns+1,ks+k-1}$ by using $G_{n,k} \to G_{n(s-s'),k(s-s')}$ and the additivity property.

Some of the earlier results related to the conjecture show that for arbitrarily chosen values n and k there exists certain subregion of the light area of Figure 1 such that for any grid point (n', k') from this subregion, $G_{n,k} \not\to G_{n',k'}$. For example, Lovász's theorem in [2] states that the chromatic number of $G_{n,k}$ is $k+2$. This is equivalent with saying that for any grid point (n', k') from the stripe determined by the lines $y = k-1$ and $y = 0$, $G_{n,k} \not\to G_{n',k'}$. By the Erdős-Ko-Radó theorem [1] for any grid point (n', k') below the line $y = \frac{k}{n}x$, $G_{n,k} \not\to G_{n',k'}$. Stahl [5] extends this result for any grid point $(n', k') \neq (ns, ks)$ on the line $y = \frac{k}{n}x$. By the above mentioned consequence of the Erdős-Ko-Radó theorem, Stahl's conjecture holds for the case when $k = 1$ and n is arbitrary. In [5], Stahl gave another proof for this case by using the fact that $G_{n,1}$ contains a $(2n+1)$-cycle, meanwhile $G_{n',k'}$ does not if (n', k') is below the line $y = \frac{k}{n}x$. Stahl also managed to prove his conjecture for $n = 2$ and $n = 3$ and arbitrary values of k in [6]. Our goal in this paper is to prove the conjecture for the case of $n = 4$ and $k = 2$. This is the smallest pair of parameters n and k for which the conjecture was open.

2. Main results

To study the existence of homomorphisms between Kneser graphs we adopt an approach presented by Walker in [7]. Let $B_{n,k}$ denote the ordered set of the subsets of the set $\{1, \ldots, 2n+k\}$ that have size at least n and at most $n+k$. We write $B_{n,k} \to B_{n',k'}$ to denote that there is a monotone map from $B_{n,k}$ to $B_{n',k'}$ that preserves disjointness. Such a disjointness preserving monotone map is called a homomorphism. Clearly, if φ is a graph homomorphism from $G_{n,k}$ to $G_{n',k'}$, then the map $A \mapsto \cup_{B \subseteq A} \varphi(B)$ is a homomorphism from $B_{n,k}$ to $B_{n',k'}$. Conversely, if ψ is a homomorphism from $B_{n,k}$ to $B_{n',k'}$, then the map that assigns an n'-element subset of $\psi(A)$ to every A in $G_{n,k}$ is a homomorphism from $G_{n,k}$ to $G_{n',k'}$. Thus, there exists a graph homomorphism from $G_{n,k}$ to $G_{n',k'}$ if and only if $B_{n,k} \to B_{n',k'}$. We shall use the latter equivalent condition for proving our main result in the paper.

In the sequel we require a result from Osztényi [3]. The distance $d(R, Q)$ between two elements R and Q of $B_{n,k}$ is the shortest length of a path between R and Q in the comparability graph of $B_{n,k}$.

Lemma 3. *Let* $n = k\lceil \frac{n}{k} \rceil - r$.

1. *min*$\{d(R, Q) :\ R, Q \in B_{n,k}$ *and* $R \cap Q = \emptyset\} = 2\lceil \frac{n}{k} \rceil$
2. *If* $R, Q \in B_{n,k}$ *and* $R \cap Q = \emptyset$, *then* $d(R, Q) = 2\lceil \frac{n}{k} \rceil$ *if and only if* $|R| + |Q| \leq 2n + r$.

The spider web is the poset of height one depicted in Figure 2. It has black nodes and white nodes. Black nodes are minimal and white nodes are maximal

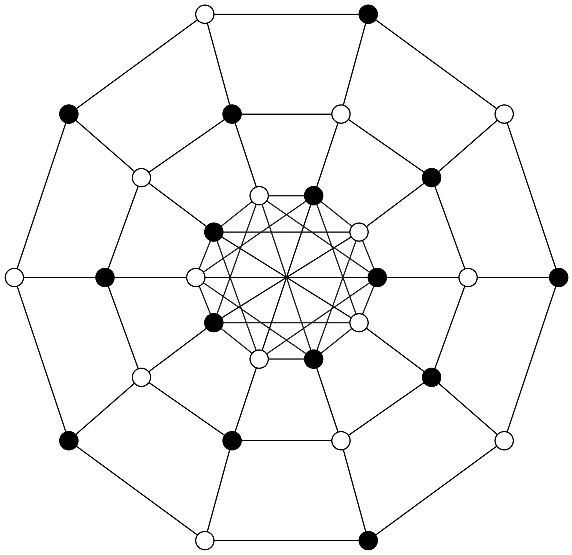

FIGURE 2. The spider web (a top view)

elements of the poset. We say that $B_{n,k}$ contains the spider web if there exists a monotone map from the spider web to $B_{n,k}$ that sends the antipodal elements on the boundary of the spider web to disjoint elements of $B_{n,k}$. In this situation the image of the spider web as a subposet is called a spider web in $B_{n,k}$. If S is a spider web in $B_{n,k}$ we shall talk about the black and white nodes of S. These are simply the images of nodes of the same respective color of the spider web under the relevant map. At this point we note that a node of a spider web in $B_{n,k}$ may have multiple colors as the map associated with it may not be injective.

Next, we shall prove Conjecture 2 for the case of $n = 4$ and $k = 2$. The idea of our proof is somewhat similar to that of Stahl's proof for the case $k = 1$ in [5]; we exhibit an obstruction of small size to the existence of a homomorphism. By Walker's approach it suffices to prove the following.

Theorem 4. *For all non-negative integers s, $B_{4,2} \not\to B_{4s+1,2s+1}$.*

Proof. As we noted earlier, it suffices to prove the claim for arbitrarily large values of s, so we assume that $s \geq 1$. We also assume that the elements of $B_{4s+1,2s+1}$ are subsets of a $(10s + 3)$-element set U. Suppose to the contrary of the claim that $B_{4,2} \to B_{4s+1,2s+1}$. Since $B_{4,2}$ contains a spider web, $B_{4s+1,2s+1}$ contains a spider web, too. Let S be a spider web in $B_{4s+1,2s+1}$. Observe that each node of S is contained in a path of length at most 4 between two disjoint black nodes of S. Let P be a path like that in $B_{4s+1,2s+1}$. By Lemma 3, P has length 4 and the end nodes of P have $4s + 1$ or $4s + 2$ elements. This is also true for the middle node of P. Indeed, if the middle node had at least $4s + 3$ elements, then the three black nodes of P would have a nontrivial intersection which would violate the disjointness of the two end nodes of P. So the black nodes of P have either $4s + 1$ or $4s + 2$ elements. Moreover, each white node of P has at least $6s + 1$ elements, for otherwise the two end nodes of P would share a common element, a contradiction.

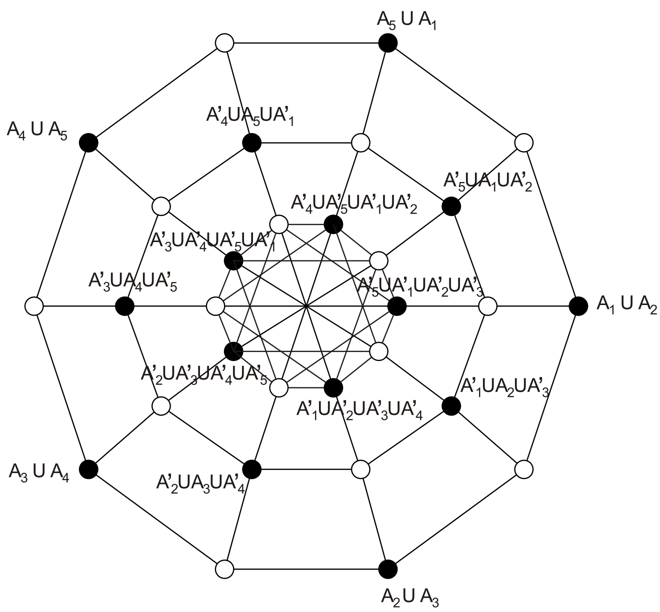

FIGURE 3. A spider web in $B_{4,2}$, where $A_i = \{2i-1, 2i\}$ and $A'_i = \{2i-1\}$ for $1 \leq i \leq 5$ and the white nodes are obtained by taking unions of neighbors.

The above bounds on the sizes of black nodes and white nodes of S show that the sets of black nodes of S is disjoint from the set of white nodes of S. Then, by possibly throwing out one element from each of the black nodes of S, we may assume that each of the black nodes of S has $4s+1$ elements. We observe that the nodes of the boundary of S are pairwise different subsets of U. Indeed, any two nodes of the boundary of S are contained in a path of length 4 between two disjoint black nodes, or one of the two nodes is a white node and the other contains the antipodal black node.

Let X, Y and Z be the consecutive minimal elements of a path of length 4 between two disjoint elements X and Z in $B_{4s+1,2s+1}$ such that $|X| = |Y| = |Z| = 4s+1$. Then we claim that $2s \leq |X \cap Y| \leq 2s+1$ and that $|X \cap Y| = |Y \cap Z| = 2s+1$ does not hold. Indeed, if $|X \cap Y| \leq 2s-1$, then we should throw out at least $2s+2$ elements from X to get Y, which is impossible to do in one step in $B_{4s+1,2s+1}$. If $2s+2 \leq |X \cap Y|$, then we should also throw out at least $2s+2$ elements from Y to get Z, which is impossible. Finally, if $|X \cap Y| = |Y \cap Z| = 2s+1$, then

$$4s+2 = |X \cap Y| + |Y \cap Z| = |(X \cup Z) \cap Y| \leq |Y| = 4s+1,$$

a contradiction.

Let $X, Y, Z, W \in S$ as displayed in Figure 4. We claim that $|X \cap Y| = |Y \cap Z| = |Z \cap W| = 2s$ does not hold. Suppose that all of these equalities hold. Then $|W \cup Z| = |W| + |Z| - |W \cap Z| = 6s+2$. Since X, Z, W are all subsets of the $(10s+3)$-element set U, $|X| = 4s+1$ and $X \cap (W \cup Z) = \emptyset$, hence $X \cup Z \cup W = U$. So $Y = (Y \cap X) \cup (Y \cap Z) \cup (Y \cap W)$. As $Y \cap W = \emptyset$, $Y = (Y \cap X) \cup (Y \cap Z)$, which implies that $|Y| = 4s$, a contradiction.

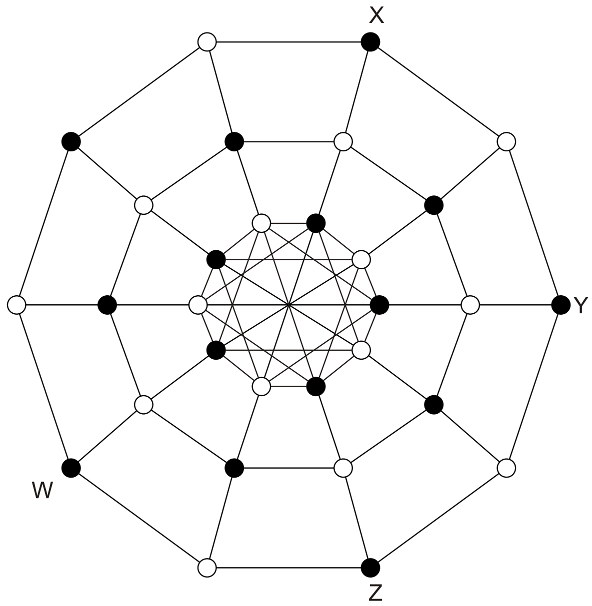

FIGURE 4

Summing up what we have so far, on the boundary of S the intersection of two consecutive black nodes has $2s$ or $2s+1$ elements and among those intersections there are no two consecutive ones of size 2s+1 and no three consecutive ones of size $2s$. According to these conditions we depicted the sizes of the intersections of consecutive black nodes on the boundary of S in Figure 5. The sizes and the disjointness relation in S yield the existence of pairwise disjoint subsets $A_1, \dots, A_5$ and E in U such that $|A_1| = |A_2| = |A_4| = 2s$, $|A_3| = |A_5| = 2s+1$, $|E| = 1$ and the black nodes on the boundary of S are of the form shown in Figure 5.

Let O be the union of the images of the five innermost black nodes of the spider web in S. Clearly, O is less than or equal to the intersection of the images of the five innermost white nodes of the spider web in S. We claim that O can be assumed to have at most $5s+1$ elements. Indeed, if O had at least $5s+2$ elements, then we could increase by one element the white nodes of $6s+1$ elements on the boundary of S in such a way that the disjointness relation would still hold and, by complementation, would get a new spider web S' in $B_{4s+1,2s+1}$ such that the union of the images of the five innermost black nodes of the spider web in S' would have at most $5s+1$ elements. So from now on we assume that $|O| \leq 5s+1$.

Since for any black node Q on the boundary of S, Q and O are contained in a path of minimum length between two disjoint black nodes on one of the diagonals of S, $2s \leq |Q \cap O| \leq 2s+1$. For $A \in \{A_1, A_2, A_3, A_4, A_5, E\}$ let $A' = A \cap O$. Thus $O = A'_1 \cup A'_2 \cup A'_3 \cup A'_4 \cup A'_5 \cup E'$, and we have the following system of equations for the sizes of a_i of A'_i and e of E'.

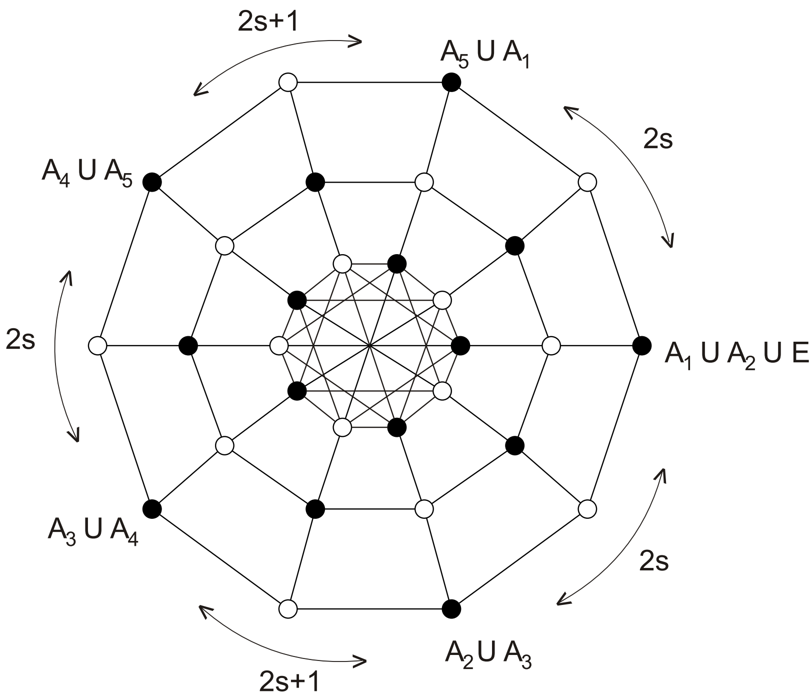

FIGURE 5. The structure of the boundary of S in $B_{4s+1,2s+1}$

$$(*)\qquad \begin{aligned} e + a_1 + a_2 &= 2s + \epsilon_1 \\ a_2 + a_3 &= 2s + \epsilon_2 \\ a_3 + a_4 &= 2s + \epsilon_3 \\ a_4 + a_5 &= 2s + \epsilon_4 \\ a_5 + a_1 &= 2s + \epsilon_5, \end{aligned}$$

where $\epsilon_i \in \{0, 1\}$.

By summing up these equations we get that

$$(**)\qquad 2|O| = 10s + \sum_{i=1}^{5} \epsilon_i + e.$$

So by $|O| \leq 5s+1$ we have that $\sum_{i=1}^{5} \epsilon_i + e \leq 2$. Hence at most two of the ϵ_i equal 1. The above equation also shows that if none or two of the ϵ_i equal 1, then $e = 0$, and if exactly one of the ϵ_i equals 1, then $e = 1$.

In Figure 6 we displayed O and some elements of S those of which we make a claim on before launching into the case by case argument of the proof. We note that some nodes of the same color in the figure may collapse, not on the boundary though.

Claim 1. If

$$|O \cap X_1| = |O \cap X_2| = 2s,$$

then

$$W \subseteq (O \cap X_1) \cup (O \cap X_2) \cup (X_1 \cap X_2) \cup (O \cap E).$$

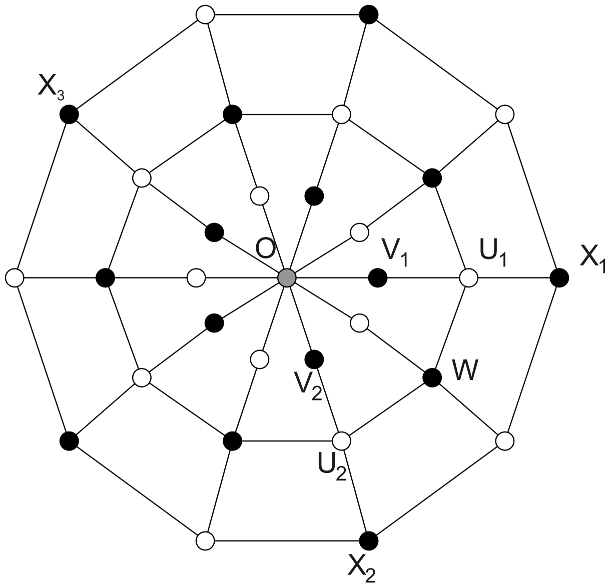

FIGURE 6

Indeed, since $V_i \subseteq O$, $V_i \cap X_i \subseteq O \cap X_i$ and $V_i \cup X_i \subseteq O \cup X_i$ for $1 \leq i \leq 2$. The equality $|O \cap X_i| = 2s$ implies $|V_i \cap X_i| = 2s$, and hence $U_i = V_i \cup X_i$, and so $U_i \subseteq O \cup X_i$ for $1 \leq i \leq 2$. Now, $W \subseteq U_1 \cap U_2 \cap \bar{X}_3$, and so

$$W \subseteq (O \cup X_1) \cap (O \cup X_2) \cap (X_1 \cup X_2 \cup E) = (O \cap X_1) \cup (O \cap X_2) \cup (X_1 \cap X_2) \cup (O \cap E).$$

Next we obtain a contradiction for all possible values of the ϵ_i, which proves that $B_{4,2} \to B_{4s+1,2s+1}$ does not hold. To follow the argument see Figure 7.

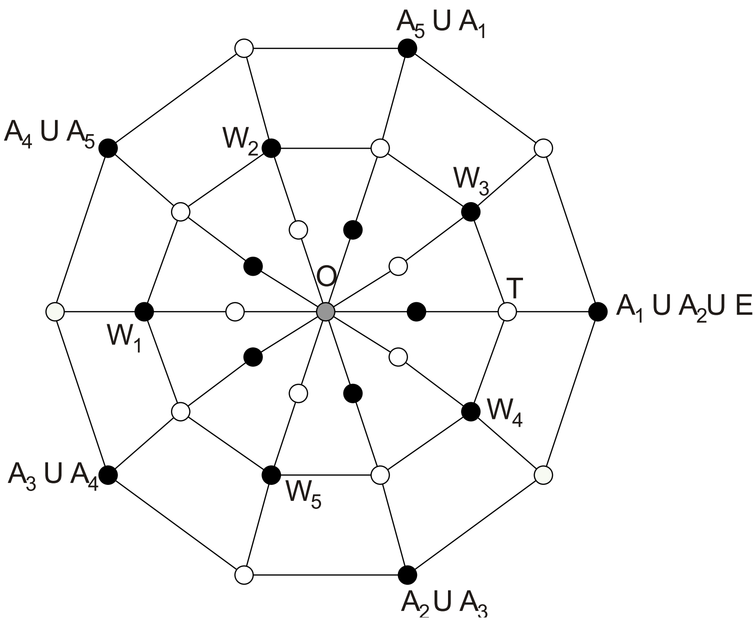

FIGURE 7

Case 1. We assume that for $1 \leq i \leq 5$, $\epsilon_i = 0$. Then we saw that $e = 0$ and from $(*)$ all of the a_i equal s. By the preceding claim we have that

$$W_1 \subseteq A'_3 \cup A_4 \cup A'_5,$$

which yields

$$|W_1| \leq |A'_3 \cup A_4 \cup A'_5| = 4s,$$

a contradiction.

Case 2. We assume that one of the ϵ_i is 1 and the others are 0. Then we saw that $e = 1$. By symmetry it suffices to consider the following three subcases.

(*i*) Suppose that $\epsilon_1 = 1$. Then from $(*)$ all of the a_i equal s and the claim applied to W_1 in the same way as in Case 1 gives a contradiction.

(*ii*) Suppose that $\epsilon_2 = 1$. Then from $(*)$ again,

$$a_1 = a_4 = s - 1,\ a_2 = s \text{ and } a_3 = a_5 = s + 1.$$

By Claim 1, $W_2 \subseteq A'_4 \cup A_5 \cup A'_1 \cup E$, which yields

$$|W_2| \leq |A'_4 \cup A_5 \cup A'_1 \cup E| = 4s,$$

a contradiction.

(*iii*) Suppose that $\epsilon_3 = 1$. Then from $(*)$,

$$a_2 = s - 1,\ a_1 = a_4 = a_5 = s \text{ and } a_3 = s + 1$$

and $W_3 \subseteq A'_5 \cup A_1 \cup A'_2 \cup E$ yields a contradiction again.

Case 3. We assume that two of the ϵ_i are 1 and the others are 0. Then we saw that $e = 0$. Up to symmetry it suffices to consider the following six subcases. In the first three of these subcases we assume that two consecutive ϵ_i equals 1.

(*i*) Suppose that $\epsilon_1 = \epsilon_2 = 1$. Then from $(*)$ we get that

$$a_1 = a_3 = a_4 = a_5 = s \text{ and } a_2 = s + 1$$

and Claim 1 applied to W_1 in the same way as in Case 1 yields a contradiction.

(*ii*) Suppose that $\epsilon_2 = \epsilon_3 = 1$. Then from $(*)$ we get that

$$a_1 = a_2 = a_4 = a_5 = s \text{ and } a_3 = s + 1$$

and $W_3 \subseteq A'_5 \cup A_1 \cup A'_2$ yields a contradiction.

(*iii*) Suppose that $\epsilon_3 = \epsilon_4 = 1$. Then from $(*)$ we get that

$$a_1 = a_2 = a_3 = a_5 = s \text{ and } a_4 = s + 1.$$

Similarly as in the preceding subcase we get a contradiction.

For the next two subcases we require a new claim whose proof is an analogue of that of Claim 1 (for notation see Figure 6).

Claim 2. If

$$|O \cap X_1| = 2s + 1,\ |O \cap X_2| = 2s \textit{ and } e = 0,$$

then

$$|W| \leq |(O \cap X_1) \cup (O \cap X_2) \cup (X_1 \cap X_2)| + 1.$$

For completeness we supply a proof. As we saw in the proof of Claim 1, the equality $|O \cap X_2| = 2s$ implies $U_2 \subseteq O \cup X_2$. Similarly, the equality $|O \cap X_1| = 2s+1$

implies that $|V_1 \cap X_2|$ is $2s$ or $2s+1$, and hence $U_1 = V_1 \cup X_1 \cup \{v\}$ for some $v \in U$, and so $U_1 \subseteq O \cup X_1 \cup \{v\}$. Now, $W \subseteq U_1 \cap U_2 \cap \bar{X}_3$, and so

$$|W| \leq |(O\cup X_1\cup\{v\})\cap(O\cup X_2)\cap(X_1\cup X_2\cup E)| \leq |(O\cap X_1)\cup(O\cap X_2)\cup(X_1\cap X_2)|+1.$$

(iv) Suppose that $\epsilon_1 = \epsilon_3 = 1$. Then

$$a_5 = s-1,\ a_2 = a_3 = s \text{ and } a_1 = a_4 = s+1$$

and by Claim 2, $|W_1| \leq |A_3' \cup A_4 \cup A_5'| + 1 = 4s$, a contradiction.

(v) Suppose that $\epsilon_3 = \epsilon_5 = 1$. Then

$$a_2 = s-1,\ a_4 = a_5 = s \text{ and } a_1 = a_3 = s+1$$

and by Claim 2, $|W_3| \leq |A_5' \cup A_1 \cup A_2'| + 1 = 4s$, a contradiction.

The only remaining subcase needs an extra argument as follows.

(vi) Suppose that $\epsilon_2 = \epsilon_5 = 1$. Then

$$a_4 = s-1,\ a_1 = a_2 = s \text{ and } a_3 = a_5 = s+1.$$

Hence $|O \cap (A_5 \cup A_1)| = 2s+1$, $|O \cap (A_4 \cup A_5)| = 2s$. So as in the proof of Claim 2,

$$W_2 \subseteq (O \cup A_5 \cup A_1 \cup \{w\}) \cap (O \cup A_4 \cup A_5) \cap (A_4 \cup A_5 \cup A_1 \cup E)$$

for some w in U. By the use of $e = 0$ we get $W_2 \subseteq A_4' \cup A_5 \cup A_1' \cup \{w\}$. Since $|A_4' \cup A_5 \cup A_1' \cup \{w\}| \leq 4s+1$, $W_2 = A_4' \cup A_5 \cup A_1' \cup \{w\}$. Then $w \in O \cup A_4 \cup A_5$. On the other hand $w \notin O \cup A_1 \cup A_5$, for otherwise $W_2 \subseteq A_4' \cup A_5 \cup A_1'$ and hence $|W_2| \leq 2s$, a contradiction. Hence $w \in A_4 \setminus A_4'$. Since T is a subset of $A_1 \cup A_2 \cup E \cup O$, $w \notin T$. Hence $w \notin W_3$. On the other hand $W_3 \subseteq A_5' \cup A_1 \cup A_2' \cup \{w\}$ and $|A_5' \cup A_1 \cup A_2'| = 4s+1$. So $W_3 = A_5' \cup A_1 \cup A_2'$. By a symmetric argument starting with W_5 yields that $W_4 = A_1' \cup A_2 \cup A_3'$. Then T contains

$$W_3 \cup W_4 \cup A_1 \cup A_2 \cup E = A_1 \cup A_2 \cup A_3' \cup A_5' \cup E.$$

But $|A_1 \cup A_2 \cup A_3' \cup A_5' \cup E| \geq 6s+3$, a contradiction.

□

3. Connections with the Schrijver graph

The Schrijver graph $S_{n,k}$ is the subgraph of $G_{n,k}$ that is spanned by the vertices with no neighboring elements modulo $2n+k$. In [4] Schrijver proved that there is no graph homomorphism from $S_{n,k}$ to $G_{1,k-1}$, which implies Lovász's result that there is no homomorphism from $G_{n,k}$ to $G_{1,k-1}$. Actually, the proof of Theorem 4 in the preceding section shows, as explained in the next paragraph, that a similar idea works for $n = 4$ and $k = 2$ and for all s, that is, there is no graph homomorphism from $S_{4,2}$ to $G_{4s+1,2s+1}$, and so there is none from $G_{4,2}$ to $G_{4s+1,2s+1}$.

Indeed, $S_{4,2}$ yields a poset $P_{4,2}$ whose base set is formed by the vertices of $S_{4,2}$ and their complements, and whose ordering is containment. We depicted this poset in Figure 8, where for the sake of clarity all edges between the black and white nodes of the boundary are left out. Our spider web is obtained from $P_{4,2}$ by removing the nodes of the boundary and the nodes adjacent to them. Note that there is a monotone disjointness preserving map from the spider web to $B_{4s+1,2s+1}$ if and only if there is one from $P_{4,2}$ to $B_{4s+1,2s+1}$ if and only if there is a graph homomorphism from $S_{4,2}$ to $G_{4s+1,2s+1}$. So by the proof of Theorem 4 there is no such a graph homomorphism.

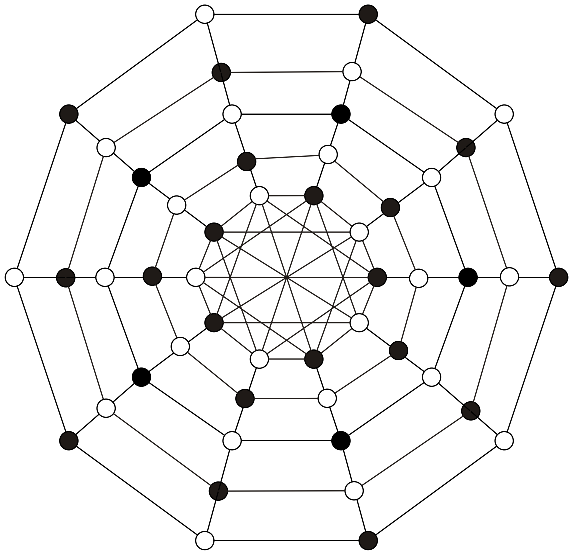

FIGURE 8. The poset $P_{4,2}$ (a top view)

Let $n > 4$. One may speculate that there still may not exist a graph homomorphism from $S_{n,2}$ to $G_{ns+1,2s+1}$, and this makes Stahl's conjecture true for n and $k = 2$. Unfortunately, for the cases $n = 5$ and $n = 6$ we have found graph embeddings from $S_{5,2}$ to $G_{6,3}$ and from $S_{6,2}$ to $G_{7,3}$ via a computer program, see the files *http://www.math.u-szeged.hu/∼makay/map5_2.txt* and *map6_2.txt*. Let $S'_{n,2}$ be the subgraph spanned by the vertices of $G_{n,2}$ that are comparable to some of the complements of the vertices of $S_{n,2}$. Clearly, $S_{n,2}$ is a subgraph of $S'_{n,2}$. It was also checked by computer that there is no homomorphism from $S'_{5,2}$ to $G_{6,3}$, which gives $G_{5,2} \not\to G_{6,3}$. Our findings for $G_{5,2}$ and $S_{5,2}$ also show that the *multichromatic numbers* (defined in [5]) of a Kneser graph and the related Schrijver graph may differ, as opposed to their usual chromatic numbers.

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Bolyai Intézet, Aradi vértanúk tere 1, H-6720, Szeged, Hungary

E-mail address: kincses@math.u-szeged.hu

E-mail address: makayg@math.u-szeged.hu

E-mail address: mmaroti@math.u-szeged.hu

E-mail address: osztenyi@math.u-szeged.hu

E-mail address: zadori@math.u-szeged.hu